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Investigation of Geomagnetic Field Forecasting and Fluid Dynamics of the Core

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1. Problems

No problems are impeding progress.

2. Approach

Two new approaches were developed during the preceding quarter (1 Jun.-31 March 1981) and described in Progress Report #5 (to which reference should be made in evaluating this report).

We now describe another new approach that has been developed during the current report period. The theoretical and practical importance of the "unsigned magnetic flux" or pole-strength of the earth, defined at radius r and time t by

$$P(r,t) = \int_0^{2\pi} \int_0^{\pi} |B_r(r,\theta,\phi,t)|\, r^2 \sin\theta \; d\theta d\phi \qquad (1)$$

stems from two considerations. Firstly, this somewhat bizarre magnetic quantity is unique in that it is rigorously invariant in time at the top of earth's liquid core provided the core moves like a perfect conductor, which it must do on sufficiently short time scale, since ohmic diffusion is not instantaneous. Secondly, the constancy of P at $r = r_c$ = 3485 km provides one constraint for the magnetic forecast problem, since a mainfield model for some future epoch should produce the same value of P at $r = r_c$.

Because of these facts we believe it important to understand P both physically and mathematically. We have therefore

developed a method for evaluating P approximately at the surface of the earth $r = a = 6371.2$ km (where P is also of interest since even though it is not constant, its time dependence is weak and smooth--see Figure 1 in section 3 below).

Expressing $\underline{B}$ as the curl of a vector potential $\underline{A}$ and then applying Stokes' theorem shows that an alternative expression for P is

$$P(r,t) = 2 \oint_C \underline{A}(r,\theta,\phi,t) \cdot d\underline{s} \ , \qquad (2)$$

where the contour integral is around each curve C on the spherical surface of radius r where the vertical magnetic field, B_r, vanishes. At the surface of the earth there is only one such curve, the magnetic equator, and it is everywhere no more than about 13° displaced from the geographic equator. Consequently, small angle approximations to trigonometirc functions can be introduced and these lead to a very great simplification of the required integration. The details would be inappropriate to give here, but the first order result is simply that

$$P(a,t) \doteq -4\pi a^2 (g_1{}^0 - \tfrac{1}{2} g_3{}^0)$$

where $g_1{}^0$, $g_3{}^0$ are the axial dipole and axial octupole Gauss coefficients. The above approximation is found to be accurate to about 0.6% by comparing its prediction with an accurate numerical calculation. A better approximation is being

developed, but it involves a fairly heavy amount of tedius algebra.

A different but simple result of beauty is obtained for the conceptually important case where the magnetic field is purely a centered (but tilted) dipole. Then in tilted dipole coordinates the value for P_d (= dipolar contribution to P) can be worked out with ease, the result being

$$P_d(r,t) = 4\pi \frac{a^3}{r} [(g_1{}^0)^2 + (g_1{}^1)^2 + (h_1{}^1)^2]^{\frac{1}{2}} . \qquad (3)$$

Stated in words: the unsigned dipole magnetic flux is an inverse function of radius proportional to the magnetic dipole moment. At earth's surface, r = a, the unsigned magnetic dipole flux per unit area is the magnetic dipole moment per unit volume. Equation (3) reveals that P_d, and therefore P, depends non-linearly on each Gauss coefficient in general. This conflicts with some recent work of Hide and Malin (1981, Proc. Roy. Soc. A374, 15-33) and may in part explain some mysterious time dependence in their calculations.

3. Accomplishments

We focus on two particular accomplishments, both involving the pole-strength as discussed above.

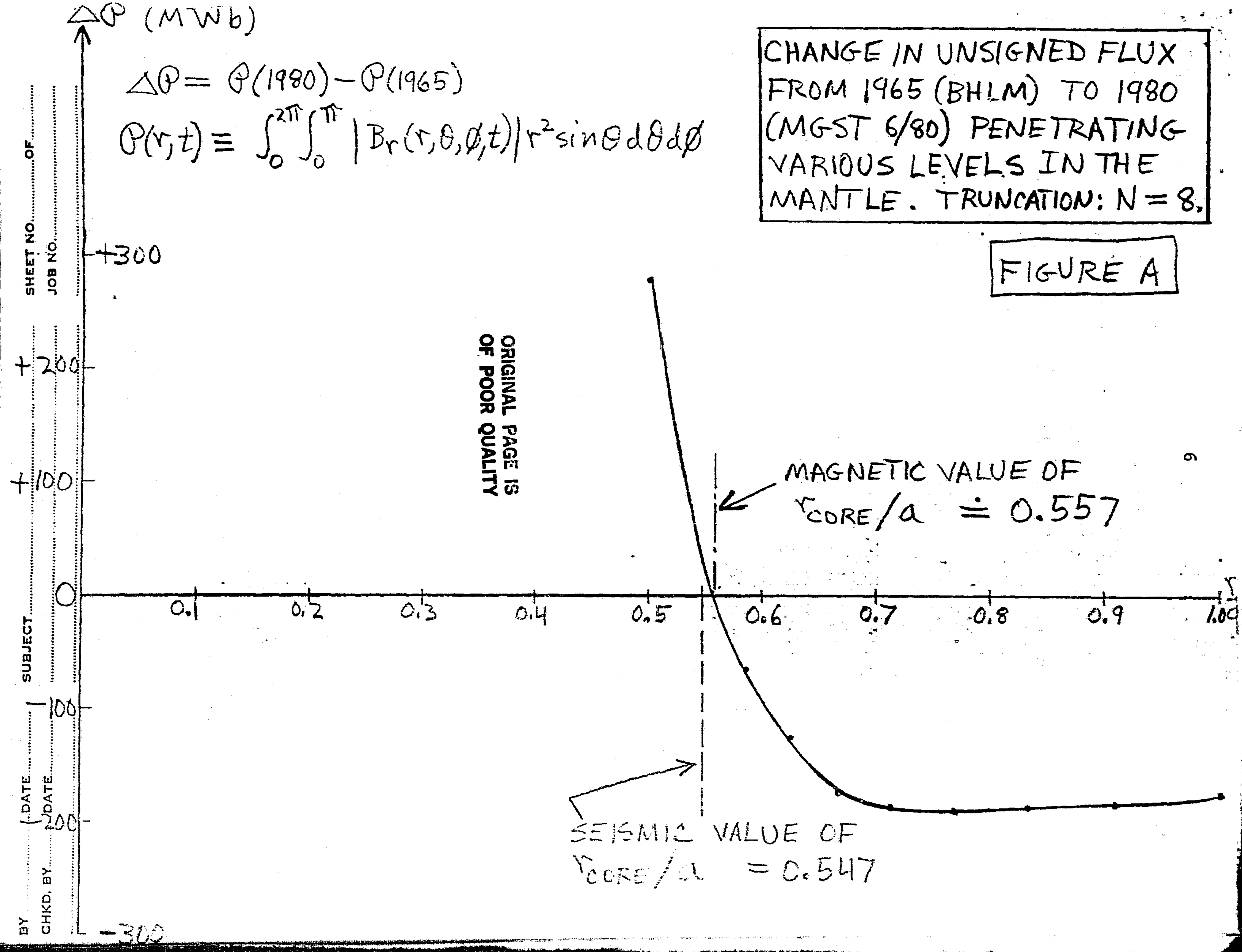
a) Magnetic determination of the depth to the core-mantle boundary using MAGSAT data.

Since P generally changes in time except at the top of earth's core, $r = r_c$, a calculation of P as a function of

depth for two different times, t_1, t_2, provides a magnetic determination of the radius of the core-mantle boundary. This idea was proposed by Hide and applied by Hide and Malin, but they used a mainfield model together with a secular variation model and obtained results which are suggestive but not definitive.

We have produced what we believe to be a definitive test of the concept by reducing the dependence on secular variation (which is highly uncertain when evaluated at depths within the earth) to negligible proportions. For t_1 we choose 1965 and adopt the so-called definitive mainfield model of Barraclough, Harwood, Leaton, and Malin. It is an interpolative rather than extrapolative model built on a massive data set centered in time near 1965. Most data used were within 5 years of this epoch so minimum reliance on a SV model was required in bringing the data to common epoch. For t_2 we choose 1980 and adopt the mainfield model from MAGSAT of Langel et al. MGST 6/80. This used data only from the two quiet days 5,6 Nov. 1980, so it too is virtually free of secular variation. This model was a fit to the data at $N = 13$, but we have truncated back to $N = 8$ since that was the value used for the 1965 model (we plan to re-do the calculation using an $N = 8$ fit to the MAGSAT data, but expect the results to change negligibly).

The primary result is plotted as Figure A. It shows how $P(r,t_2)-P(r,t_1)$ varies with depth in the mantle (assumed



CHANGE IN UNSIGNED FLUX FROM 1965 (BHLM) TO 1980 (MGST 6/80) PENETRATING VARIOUS LEVELS IN THE MANTLE. TRUNCATION: N = 8.

FIGURE A

insulating--which we believe is a sound approximation for mainfield models, but not so for secular variation). Clearly, between 1965 and 1980 throughout most of the mantle P decreased, but just at $r = 0.557\,a = 3548.8$ km ($= 3585+63.8$ km), ΔP passed cleanly through zero. Thus, this method has found the core radius magnetically to within 1.83% of its accurate seismic value. The result is highly significant in our view.

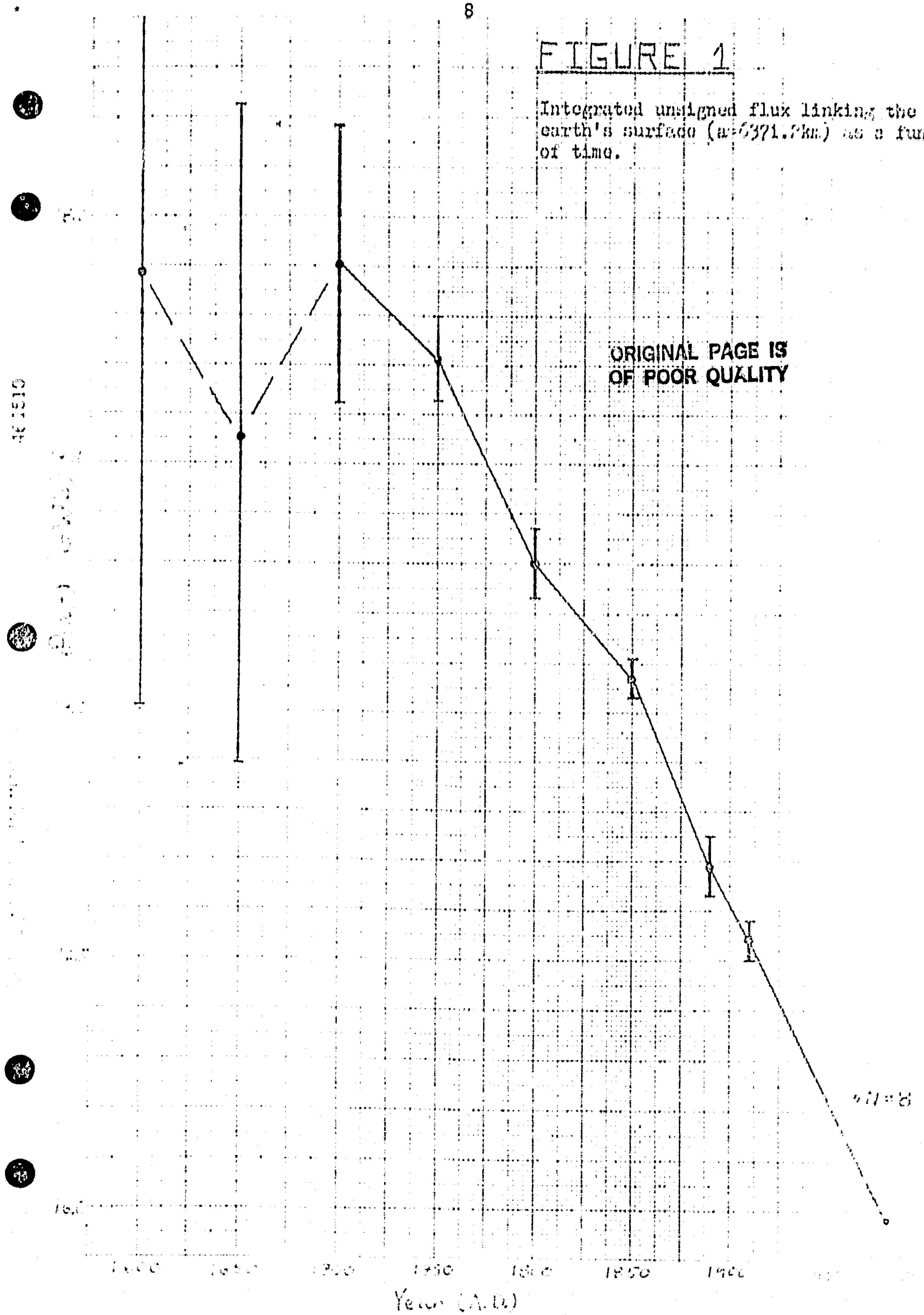
b) <u>Historical time-dependence of the unsigned magnetic flux crossing earth's surface.</u>

As planned in the last progress report, we have now completed numerical evaluation of P as a function of time for $r = a = 6371.2$ km. At earth's surface, the higher harmonics of the geomagnetic field do not contribute strongly to the value of P (as they do at earth's core). Therefore, it is meaningful to use historical models of the geomagnetic field from times back a century or more ago, for which only the fairly low order harmonics are available.

Figure 1 shows the time dependence of $P(a,t)$ using the $N = 4$ models constructed by Barraclough (1974) for 50 year intervals back to 1600. One standard deviation error bars are also included. Evidently for about the last 200 years, $P(a,t)$ has been declining steadily, smoothly and nearly linearly. It would make sense to predict that at 1985, $P(a,t)$ will be about 1.5 8 GWb (for $N = 4$). (The dot on this figure at $N = 8$ shows the value calculated from MGST 6/80,

FIGURE 1

Integrated unsigned flux linking the earth's surface (a=6371.2km) as a function of time.



thereby indicating the relatively small contribution of the higher order harmonics).

4. Significant Results

MAGSAT data has been used, in conjunction with a high quality mainfield model for epoch 1965, to confirm that the radius of earth's core-mantle boundary can indeed the found magnetically with impressive accuracy. The unsigned flux linking earth's core and mantle should therefore be a legitimate invariant for a span of time. The value from MAGSAT of this constant is 16.056 GWb (gigiwebers).

5. Publications

(Related to this project, but not directly supported by it.)

Edward R. Benton, "A simple model for determining the vertical growth rate of vertical motion at the top of earth's outer core," Physics of the Earth and Planetary Interiors 24 (1981), 242-244.

Lorant A. Muth and Edward R. Benton, "On the frozen flux velocity field at the surface of earth's core necessary to account for the poloidal main magnetic field and its secular variation," Physics of the Earth and Planetary Interiors 24 (1981), 245-252.

6. <u>Recommendations</u>

None.

7. <u>Funds Expended Through May 31, 1981</u>:

8. <u>Data Utility</u>

It should be apparent that the MAGSAT data as acquired and modelled is of very great utility to this project.